Click here to see what your answer should look like.

1 Used in annual compounding

$100 earns 5% interest compounding annually. How much is the compounded value in four years?

Compounded value in four years:	121.5506

How much is the compounded value in four and a half years?

Compounded value in four and a half years:	124.5523

2 Determining interest rate to double investment in ten years

What nominal interest rate (semi-annual compounding) is required to double an investment in ten years?

Rate required:	7.05%

3 Compounding value vs compounding frequency

Continuous compounding formulae usually contain "e" - which is approximately 2.71828. This example shows that when the compounding frequency is increased the result converges to "e".

An investment of $1 earns a nominal annual interest rate of 100%. The investment is for a term of one year. Interest compounds "n" times per year. The value of the investment at the end of one year is "v". Complete the following table showing v as a function of n.

n	v
1	2
2	2.25
4	2.441406
12	2.613035
365	2.714567
10000	2.718146
1000000	2.71828

	C	D	E	F	G	H	I	J	K	L
1		Click here to see what your answer should look like.								
2	1	Time to compound annually to a given value								
3										
4		$100 compounds annually at 6%. How long will it take to compound								
5		to a value of $120?								
6										
7		Time to compound (years):			3.128968					
8										
9	2	Time to compound semi-annually to a given value								
10										
11		$100 compounds semi-annually at a nominal annual rate of 6%.								
12		How long will it take to compound to a value of $120?								
13										
14		Time to compound (years):			3.084048					
15										
16										
17					www.tykoh.com					

Click here to see what your answer should look like.

1 Exponential function - Used with continuously compounding interest rate.

What would $100 compound to if the nominal annual continuously compounding rate is 6% and the term is one year?

Value in one year:	106.1837

What is the annual effective compounding rate?

Annual effective rate:	6.184%

2 Exponential function - Converting continuous rate to quarterly compounding.

What nominal quarterly compounding rate is equivalent to a continuously compounding rate of 7%?

Quarterly rate:	7.062%

3 Exponential function - Showing e^x is approximately 1+x if x is small.

Complete the following table

x	e^x	1+x
0.1	1.105171	1.1
0.01	1.01005	1.01
0.001	1.001001	1.001
0.0001	1.0001	1.0001

	C	D	E	F	G	H	I	J	K	L
1		Click here to see what your answer should look like.								
2	1	Ln function - Converting from periodic interest rate to continuous compounding								
3										
4		What continuously compounding rate is equivalent to an annually compounding								
5		rate of 6%?								
6										
7		Continuous rate:		5.827%						
8										
9										
10										
11		www.tykoh.com								

Click here to see what your answer should look like.

Growth may seem a simple term but it encompasses some important and subtle concepts - For example, a +10% growth followed by -10% growth leads to a different outcome than does zero growth. In this section we review and apply various measures of growth and show how these measures relate to each other.

1 Calculating arithmetic and compound average growth rate - when growth is constant

Consider the investment shown below. The value of the investment at the end of each year is listed. Find the per annum (p.a.) growth rate in each year.

Year	Value	Growth p.a.
0	50.00	
1	51.25	2.50%
2	52.53	2.50%
3	53.84	2.50%
4	55.19	2.50%
5	56.57	2.50%
6	57.98	2.50%

Determine the average (mean) yearly growth rate.

Mean growth rate:	2.50%

Calculate the compound average growth (CAGR) rate.

CAGR	2.50%

2 Calculating arithmetic and compound average growth rate - when growth varies

Consider the investment shown below. The value of the investment at the end of each year is listed. Find the p.a. growth rate in each year.

Year	Value	Growth p.a.
0	50.00	
1	50.00	0.00%
2	49.00	-2.00%
3	52.00	6.12%
4	51.00	-1.92%
5	53.00	3.92%
6	55.00	3.77%

Determine the average (mean) yearly growth rate.

Mean growth rate:	1.65%

Calculate the compound average growth (CAGR) rate.

CAGR	1.60%

Note the difference between the average short term growth rate in cell F47 and the long term growth rate (CAGR) in cell F51. In this example you should find that the CAGR is less than the average arithmetic growth rate. Can CAGR ever be greater?

3 Calculating quarterly, semi and p.a. growth given monthly growth

Monthly growth is 1%. What is the growth in one quarter? Six months? One year?

Period		Growth
One quarter	[%]	3.03%
Six months	[%]	6.15%
One year	[%]	12.68%

4 Calculating effective growth rate given the nominal growth rate

The nominal rate of growth p.a. is 10%. Every six months the underlying increases by half of 10% (i.e. by 5%). What is the effective yearly growth rate?

Effective rate	[%]	10.25%

5 Calculating yearly effective growth rate given nominal continuously compounding rate

The nominal continuously compounding rate is 10%. What is the effective yearly growth rate?

Effective rate	[%]	10.52%

6 Calculating nominal growth rate given the effective growth rate

The nominal rate of growth p.a. is N (%). Every quarter the underlying increases by one quarter of N. In one year the underlying grows by 12%. What is the nominal rate of growth on a quarterly basis (i.e, what is N)?

Nominal rate	[%]	11.49%

7 Calculating nominal compounding rate given yearly effective rate.

The yearly effective rate is 10%. What is the equivalent continuously compounding nominal rate?

Nominal rate	[%]	9.53%

8 Showing that 10% growth followed by -10% growth has a different result than has zero growth.

An investment has an initial value of 100. In the first year its value changes by +10% and in the second year by -10%. What is the final value of the investment?

Final value	99

What compound average growth rate (CAGR) is this equivalent to?

CAGR	-0.50%

The CAGR you obtain above should be approximately equal to $-(10\%^{2)}/2$. Confirm that is so.

$-(10\%)^2/2$	-0.50%

	C	D	E	F	G	H	I	J	K	L	M
109											
110		If we think of the +10% and -10% growths as exhibiting volatility then we can see that volatility can "feed through" into growth by a factor of $-\sigma^2/2$. That is why many formulae relating to the evolution of asset prices and formulae relating to option pricing contain that term in the "growth" part of the formula.									
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Click here to see what your answer should look like.

1 Future value of an investment earning simple interest.

A $100 investment earns simple interest at the rate of 5% per annum (p.a.)
What is the investment's value after six months?

Investment value at six months	[$]	102.5

2 Present value of an investment earning simple interest.

An investment earning 6% per annum simple interest will have a value of $150 in three months. What is the inverstment's current value?

Investment's current value	[$]	147.7833

3 Simple interest rate as a function of present value, future value and investment term.

An investment of $150 will have a value of $163 in one and a half years. The investment earns simple interest. What is the per annum interest rate?

Interest rate	[%]	5.78%

4 Investment term as a function of present value, future value and interest rate.

An investment of $175 earns simple interest of 5.5%. In how many years will the investment be worth $185?

Years:	1.038961

Click here to see what your answer should look like.

1 Future value of an investment earning compound interest.

A $100 investment earns compound interest at the rate of 5% per annum (p.a.) What is the investment's value after six months?

Investment value at six months	[$]	102.4695

2 Present value of an investment earning compound interest.

An investment earning 6% per annum compound interest will have a value of $150 in three months. What is the inverstment's current value?

Investment's current value	[$]	147.8308

3 Compound interest rate as a function of present value, future value and investment term.

An investment of $150 will have a value of $163 in one and a half years. The investment earns compound interest. What is the per annum interest rate?

Interest rate	[%]	5.70%

4 Investment term as a function of present value, future value and interest rate.

An investment of $175 earns compound interest of 5.5% p.a. In how many years will the investment be worth $185?

Years:	1.037898

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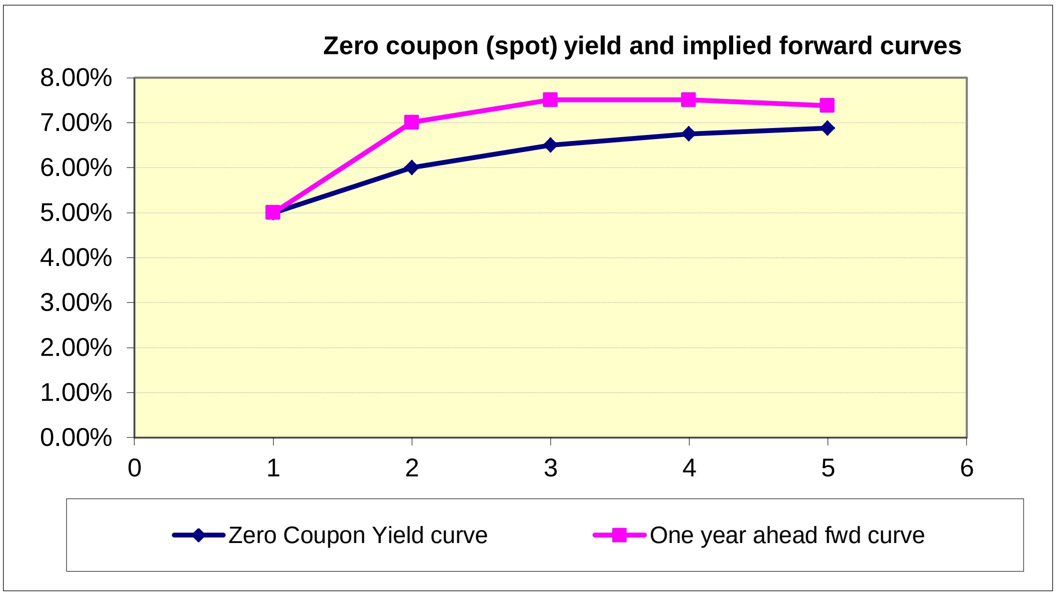
	C	D	E	F	G	H	I	J	K	L
1		Click here to see what your answer should look like.								
2	1	Future value of an investment earning continuous interest								
3										
4		An investment of $100 earns a continuous interest rate of 5%. What will the								
5		investment compound to after nine months?								
6										
7		Investment value at six months			[$]	103.8212				
8										
9	2	Present value of an investment earning continuous interest								
10										
11		An investment earning 6% per annum continuous interest will have a value of $150 in								
12		three months. What is the inverstment's current value?								
13										
14		Investment's current value			[$]	147.7668				
15										
16	3	Continuous interest rate as a function of present value, future value and investment term.								
17										
18		An investment of $150 will have a value of $163 in one and a half years. The								
19		investment earns continuous interest. What is the per annum interest rate?								
20										
21		Continuous interest rate			[%]	5.54%				
22										
23	4	Investment term as a function of present value, future value and interest rate.								
24										
25		An investment of $175 earns continuous interest of 5.5%. In how many								
26		years will the investment be worth $185?								
27										
28		Years:	1.010361							
29										
30										
31										
32					www.tykoh.com					

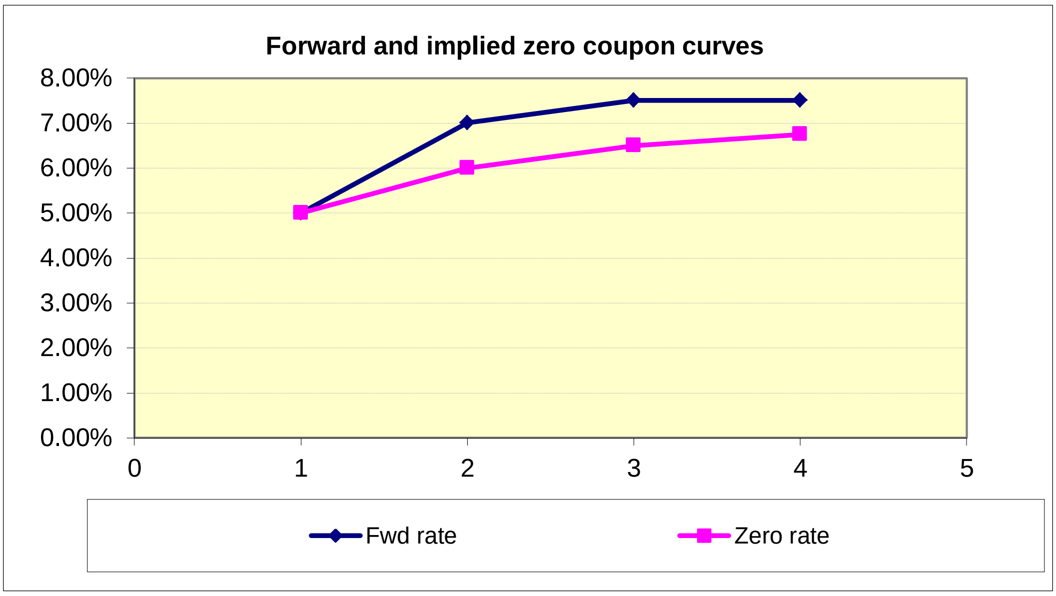
	G	H	I	J	K	L	M	N	O	P
1		Click here to see what your answer should look like.								
2	1	Calculating discount factors from points on a yield curve								
3										
4		Consider the spot / zero-coupon yield curve shown below. Yields are annual effective.								
5		Calculate discount factors corresponding to each point on the curve.								
6										
7		Time		[yrs]	0.25	0.5	0.75	1	1.25	
8		Yield p.a.		[%]	5.50%	5.70%	5.75%	5.80%	5.81%	
9		Discount factor		[#]	0.9867	0.9727	0.9589	0.9452	0.9318	
10										
11	2	Using discount factors to calculate the value of a series of future cash flows.								
12										
13		Use the discount factors above to value a security that has cash flows as shown below.								
14										
15		Time		[yrs]	0.25	0.5	0.75	1	1.25	
16		Cash flows		[$'000]	5	5	5	5	105	
17		Present value		[$'000]	4.9335	4.8633	4.7947	4.7259	97.8433	
18		Total value		[$'000]	117.1607					
19										
20	3	Generating a yield curve equivalent to a set of discount factors.								
21										
22		Consider the discount factors shown below. Calculate the equivalent spot / zero-coupon								
23		yields on an annual effective basis.								
24										
25		Time		[yrs]	0.25	0.5	0.75	1	1.25	
26		Discount factor		[#]	0.9867	0.9727	0.9589	0.9452	0.9318	
27		Yield - annual effective		[%]	5.50%	5.70%	5.75%	5.80%	5.81%	
28										
29										
30										
31										

	G	H	I	J	K	L	M	N	O	P
1		Click here to see what your answer should look like.								
2	1	Calculating forward interest rate implied by zero coupon yield curve								
3										
4		Consider the zero coupon yield curve shown below.								
5										
6		Time		[yrs]	1	2	3	4	5	
7		Yield p.a.		[%]	5.00%	6.00%	6.50%	6.75%	6.88%	
8										
9		$100 is invested at time 0 for one year. Call this investment "A". How much will								
10		the investment be worth at the end of the year?								
11										
12		Investment in one year			[$]	105				
13										
14		$100 is invested at time 0 for two years. Call this investment "B". How much will								
15		the investment be worth in two years?								
16										
17		Investment in two years			[$]	112.36				
18										
19		Investment A is re-invested in one year for an additional year. If, at maturity, the investment								
20		is worth the same as investment "B" then at what rate would the re-investment have been								
21		made?								
22										
23		Reinvestment rate		[%]	7.010%					
24										
25		The rate you have obtained is the current implied forward rate from 1 year to 2 years.								
26										
27	2	Calculating implied forward curve from yield curve.								
28										
29		In a similar way to that used above calculate the complete one-year-ahead forward curve.								
30										
31		Time t		[yrs]	1	2	3	4	5	
32		Value of $100 invested for t		[$]	105	112.36	120.795	129.8588	139.4378	
33										
34										
35			Forward rate	From	0	1	2	3	4	
36				To:	1	2	3	4	5	
37				Value:	5.00%	7.01%	7.51%	7.50%	7.38%	



3 Calculating zero curve from forward curve

Term	[yrs]		1	2	3	4
Fwd rate	[%]		5.00%	7.01%	7.51%	7.50%
Discount factor	[#]	1.0000	0.9524	0.8900	0.8278	0.7701
Zero rate	[%]		5.00%	6.00%	6.50%	6.75%



Click here to see what your answer should look like.

1 Value of a growing annuity

An annuity of $100 will be received in one year's time. Thereafter the annuity will grow at 2.0% per year. The discount rate r (annual effective) is 4.5% per year. Calculate the future and present values of the annuities and the total present value.

g	[%]	2.0%
r	[%]	4.5%
C	[$]	100

Time	[yrs]		1	2	3	4	5	6
Annuity	[$]		100.00	102.00	104.04	106.12	108.24	110.41

PV	[$]		95.69	93.40	91.17	88.99	86.86	84.78
Total PV	[$]	540.8987						

2 Present value of a basis point (PVBP)

The present value of a basis point (PVBP) is the present value of a one basis point p.a. annuity.

Nominal annual discount rate	[%]	7%
Compounding periods per year	[#]	2
Number of years	[#]	3
Per-period discount rate	[%]	3.5%

A formula can be used to calculate the PVBP. Let C be the value **(per annum)** of each cash flow, r be the nominal annual discount rate, m be the number of compounding periods per year and n be the number of compounding periods. Then the present value of the annuity is given by ..

$$PV = \frac{C}{r}\left[1 - \left(\frac{1}{1+\frac{r}{m}}\right)^{n}\right]$$

A proof of this formula is available at the following link
Annuity formula

Use the formula above to calculate the PVBP.

PVBP	[bps]	2.66

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Click here to see what your answer should look like.

1 Present value of a bond - calculating from future cash flows and a discount rate.

This exercise illustrates the fundamentals of bond valuation: Taking the future coupon and principal payments and present-valuing them. This method gives the total bond value (including accrued interest).

A bond has cash flows as shown below. The bond has a coupon rate of 6%.

Time of cash flow	[yrs]	1	2	3	4
Amount of cash flow	[$ 000's]	6	6	6	106

Determine the present value of the invidual cashflows assuming the bond's yield is 4%.

Discounted value	[$ 000's]	5.769231	5.547337	5.333978	90.60924
Total present value	[$ 000's]	107.2598			

If we buy the above bond for the amount calculated in cell K16 then the implied yield is 4%.

2 Using present value and yield to generate future cash flows.

The bond yield relates the present value of the bond to its future payments. The yield is the fixed/constant discount rate that - applied to the future cash flows - gives the present value. Another way of thinking about the yield is illustrated in this exercise: If you sell a bond, deposit the sale proceeds into an account earning the yield, then you can exactly reproduce the coupon and principal payments the bond holder requires.

If we earn 4% on investments and we start with the amount calculated in cell K16 above then we can reproduce the cash flows shown in cells K10:N11.

Opening balance	[$ 000's]	107.2598	105.5502	103.7722	101.9231
Interest earned	[$ 000's]	4.2904	4.2220	4.1509	4.0769
Coupon / principal paid	[$ 000's]	-6	-6	-6	-106
Closing balance	[$ 000's]	105.5502	103.7722	101.9231	0

So if we can lend funds at a rate of 4% we can sell a bond for the amount calculated in cell K16 and can deliver to the bond holder the coupon and principal cashflows they require.

3 Calculating yield to maturity of a bond.

Term	[yrs]		1	2	3	4
Zero rate	[%]		5.00%	6.00%	6.50%	6.75%
Fwd rate	[%]		5.000%	7.010%	7.507%	7.504%
Discount factor	[#]	1	0.952381	0.889996	0.827849	0.770067
Cashflow	[000's]		3	3	3	103

Present value	[000's]	2.8571	2.6700	2.4835	79.3169
Total PV	[000's]	87.3276			

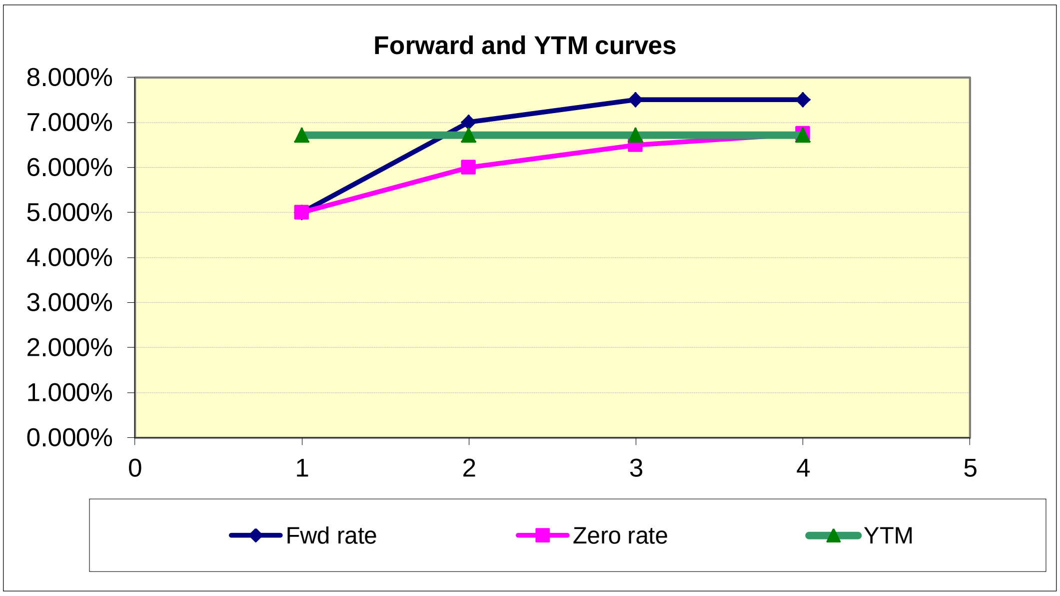
YTM	[%]	6.72%	<- Use RATE function

Check of PV using YTM

Discount factor using YTM	[#]	0.9371	0.8781	0.8228	0.7710

Present value	[000's]	2.8112	2.6342	2.4684	79.4138
Total PV	[000's]	87.3276			

The PV's in cell L50 and L59 should match.



4 Calculating yield to maturity of an annuity

Find individual and total PV's of annuities. Use the interest rates in row 44 above.

Term	[yrs]		1	2	3	4
Cashflow	[000's]		1	1	1	1
Present value	[000's]		0.9524	0.8900	0.8278	0.7701
Total PV	[000's]		3.4403			

Assemble a cash flow that alows you to use the IRR function to determine the yield to maturity. The first cash flow is minus of PV in cell L90. Subsequent values are the cash flows in row 88.

IRR cash flows	[000's]	-3.4403	1.0000	1.0000	1.0000	1.0000
YTM [IRR of cashflows in row 96].	[%]	6.31%				

Check that discounting cash flows in row 88 at YTM in cell K97 gives the same PV as in cell L90.

Cashflow in row 88 discounted at YTM.	[000's]		0.9406	0.8847	0.8322	0.7828
Total PV		3.4403				

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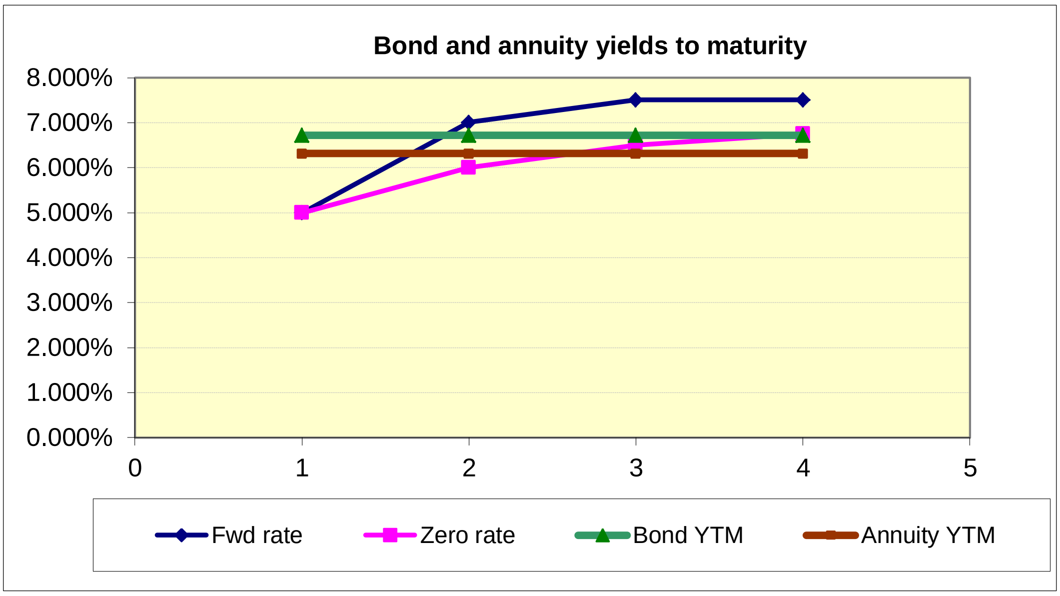


Click here to see what your answer should look like.

1 Valuing a floating rate note from first principles.

Value a floating rate note immediately after a coupon has been paid. The next coupon to be paid will have been rateset to the value shown in cell J14. Following coupons (i.e. at times 0.5, 0.75 and 1.0 years) will 'float'.

Notional	[$]	100	
Payments per year	[#]	4	
Nominal interest margin p.a.	[%]	1.00%	<- Need to add this to fwd rate when calculating FRN payment.
Nominal traded margin p.a.	[%]	10.00%	<- Need to add this to fwd rate when discounting to present value.
Next coupon rate	[%]	4.10%	

Term	[yrs]	0.00	0.25	0.50	0.75	1.00
Fwd rate p.a.	[%]		4.00%	4.50%	5.00%	5.50%
Discount factor	[#]	1.0000	0.9662	0.9324	0.8987	0.8652
FRN payment	[$]		1.2750	1.3750	1.5000	101.6250
Present value of payment	[$]		1.2319	1.2820	1.3480	87.9218
FRN value = Total PV	[$]	91.7837				

Click here to see what your answer should look like.

In this section we show how to value the fixed and floating legs of a vanilla interest rate swap so as to make it "fair" - i.e. so the net present value is zero.

1 Valuing the fixed and floating legs of a fixed-to-float interest rate swap

Receive fixed, pay float, semi-annual payments. [Represent inflows as positive and outflows as -ve.]

Notional	[$ 000's]	1,000
Fixed rate coupons	[%]	8.0000%

Term	[yrs]	0	0.5	1.0	1.5	2.0	
Fwd rate	[%]		7.00%	7.20%	7.10%	6.90%	
Discount factor	[#]	1.00000	0.96618	0.93261	0.90064	0.87060	
Floating cash flow	[$ 000's]	1,000.00	-35.00	-36.00	-35.50	-1,034.50	Total
Fixed cash flow	[$ 000's]	-1,000.00	40.00	40.00	40.00	1,040.00	[$ 000's]
Present value of fixed cash flow	[$ 000's]	-1,000.00	38.65	37.30	36.03	905.43	17.40
Present value of floating cash flow	[$ 000's]	1,000.00	-33.82	-33.57	-31.97	-900.64	0.00
PVBP	[$ 000's]	0.0000	0.0483	0.0466	0.0450	0.0435	0.18

The present value of the floating leg should be zero. Why is this so?

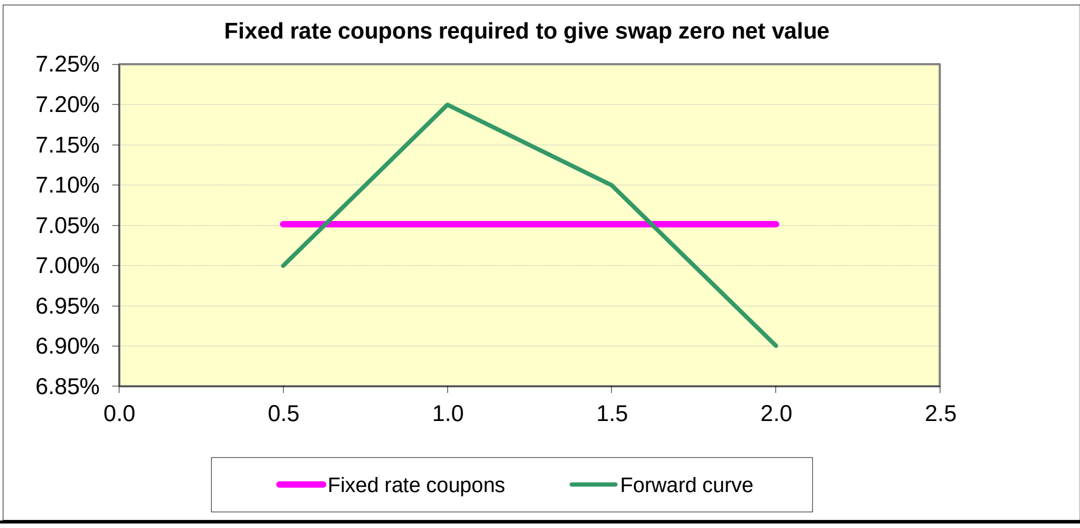
2 Calculating fixed coupon rate that makes swap value zero.

Continuing on from the preceding question: What fixed coupon rate is required in order to set the overall value of the swap (i.e. fixed leg + floating leg) to zero?

Fixed coupon rate to make swap zero value	[%]	7.0516%

Using the fixed coupon rate calculated above in cell N29:

Fixed payment	[$ 000's]	-1,000.00	35.26	35.26	35.26	1,035.26	Total
Present value of fixed payment	[$ 000's]	-1,000.00	34.07	32.88	31.75	901.30	0.00
Net cash flow		0.00	0.26	-0.74	-0.24	0.76	
PV of net cash flow		0	0.249475	-0.6918	-0.217768	0.660096	0.00



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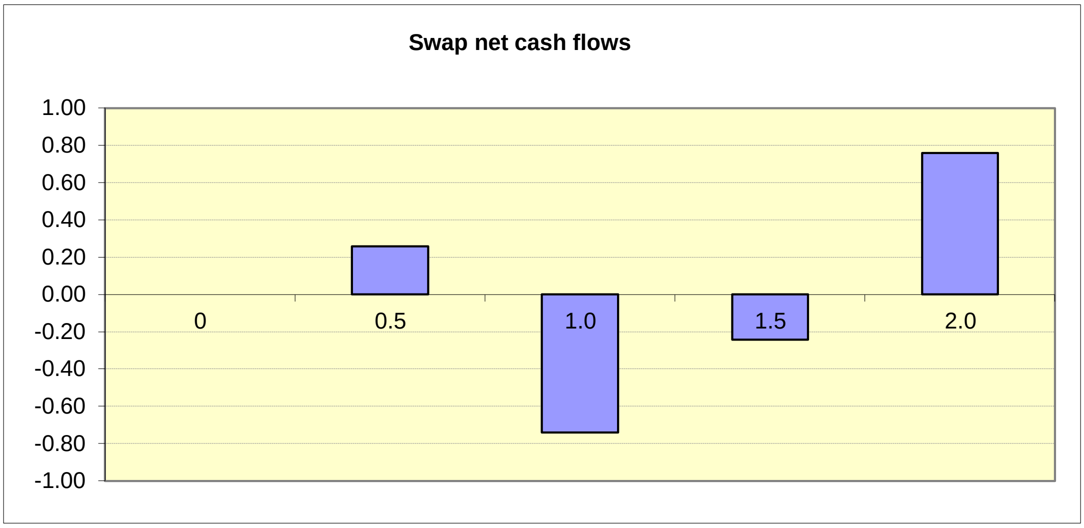


Click here to see what your answer should look like.

In this section we look at cross-currency fixed-for-floating swaps. We deal with two currencies, two discount rates and two payment frequencies and calculate the margin needed on the floating leg to make the swap "fair".

Company A issues three-year debt in USD at a fixed rate of 4.5% p.a. semi against a market swap rate of 4.0% p.a. quarterly. Company A wants floating rate AUD debt. AUD swap rates are 6.00% p.a. quarterly. What is the margin against BBSW that Company A will pay?

Bank XYZ		Company A		USD Bond Market
Swap rate (AUD, 3-yr, qtly) = 6.0% p.a.	AUD BBSW + ?% quarterly <------------------------------------		USD 4.5% semi -------------------->	Swap rate (USD, 3-yr, qtly) = 4.0% p.a.
	USD 4.5% semi ------------------------------------			

USD bond coupon (semi)	4.5%
USD quarterly 3-yr swap rate:	4.0%
AUD quarterly 3-yr swap rate	6.0%
AUD / USD exchange rate:	0.90 (i.e. 1 AUD = 0.90 USD)

1 Converting USD quarterly swap rate to semi.

Since Company A's obligation is to provide semi-annual payments into the USD Bond market we need to express rates and values on a semi-annual basis. Begin by expressing the quarterly USD swap rate on a semi-annual basis.

USD semi 3-yr swap rate:	4.0200%

2 Calculating USD PVBP

Company A has promised its USD bond holders 4.5% coupons. However, the market USD swap rate is less than this. So Company A will need to pay a margin on top of BBSW to Bank XYZ in order to receive the full 4.5%. Calculate the present value of a one basis point margin.

[PVBP - the present value of a basis point is described in the Bonds and FRNs modules in the section on annuities.]

USD semi PVBP	2.79977

3 Calculating the margin above the USD semi swap rate we want to receive

Calculate the margin above the USD semi swap rate (in cell P26) that we want to receive.

USD margin	0.4800%

4 Calculating AUD PVBP

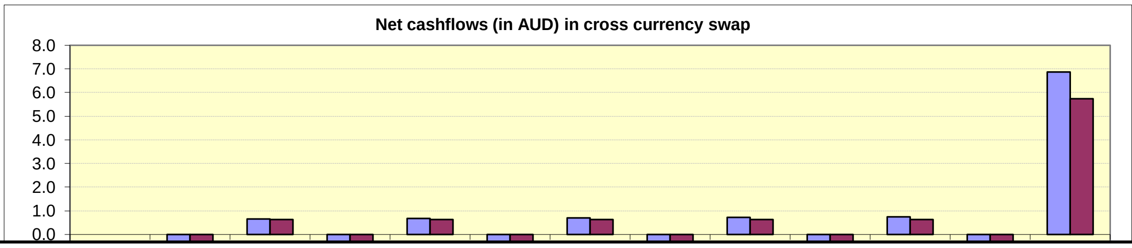
AUD quarterly PVBP	2.72688

5 Calculating AUD margin

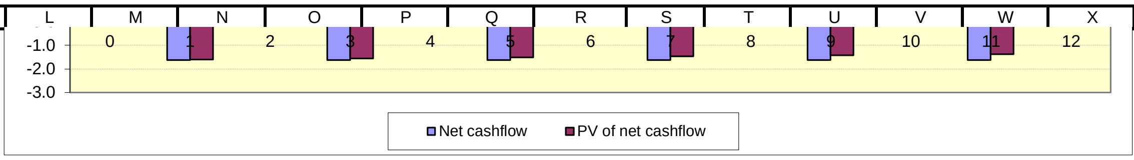
AUD margin	0.4928%

6 Modelling swap cash flows

		AUD				USD				FX rate (base 1)	Net cashflow (in AUD)	
Period	Yrs	BBSW	Discount factor	Floating payment	Present value	LIBOR	Discount factor	Fixed payment	Present value		Net cashflow	PV of net cashflow
0	0		1.00000	100.000	100.000		1.00000	-90.000	-90.000	1.000	0.000	0.000
1	0.25	6.00%	0.98522	-1.623	-1.599					0.995	-1.623	-1.599
2	0.5	6.00%	0.97066	-1.623	-1.576	4.020%	0.98030	2.025	1.985	0.990	0.649	0.630
3	0.75	6.00%	0.95632	-1.623	-1.552					0.985	-1.623	-1.552
4	1	6.00%	0.94218	-1.623	-1.529	4.020%	0.96098	2.025	1.946	0.980	0.672	0.633
5	1.25	6.00%	0.92826	-1.623	-1.507					0.976	-1.623	-1.507
6	1.5	6.00%	0.91454	-1.623	-1.484	4.020%	0.94205	2.025	1.908	0.971	0.694	0.635
7	1.75	6.00%	0.90103	-1.623	-1.463					0.966	-1.623	-1.463
8	2	6.00%	0.88771	-1.623	-1.441	4.020%	0.92348	2.025	1.870	0.961	0.717	0.637
9	2.25	6.00%	0.87459	-1.623	-1.420					0.957	-1.623	-1.420
10	2.5	6.00%	0.86167	-1.623	-1.399	4.020%	0.90529	2.025	1.833	0.952	0.741	0.638
11	2.75	6.00%	0.84893	-1.623	-1.378					0.947	-1.623	-1.378
12	3	6.00%	0.83639	-101.623	-84.996	4.020%	0.88745	92.025	81.668	0.942	6.869	5.745
		Total of AUD leg:			-1.344							
		Total of USD leg (in U69) in AUD:			1.344			Total:	1.210			
		Total of AUD & USD legs:			0.000							0.000



	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y



We can see that the net present value of the fixed and floating legs in cell X70 is zero as we would expect.

Click here to see what your answer should look like.

The sensitivity of an "output" to its "inputs" is often measured by varying the input by a small amount, determining the resultant change in output and then dividing the change in output by the change in input. "Inputs" could be: Time elapsed, time to maturity, asset price, volatility, interest rate, yield, correlation, inflation rate, fx rate, default rate and so on. "Outputs" could be value, rate of return, frequency, probability, duration and so on.

1 Calculating sensitivity of fixed-interest investment's value to time

An investment of $100 earns 6% interest compounding semi-annually. At what rate is the investment growing in value at time t = 1.5 years?

Investment value at time 1.5 years:	[$]	109.2727
Investment value at time 1.5001 years:	[$]	109.2733

Rate of increase at time 1.5 years:	[$ / yr]	6.460

What continuously compounding rate is equivalent to the semi-annual rate given above?

Continuously compounding rate:	[%]	5.912%

From your knowledge of the slope of the exponential function what is the exact answer to the question above (whose approximate answer was calculated in cell M16?

Rate of increase at time 1.5 years:	[$ / yr]	6.460

2 Calculating sensitivity of forward price to its drivers.

An asset has a spot price of S and yields a continuous return of d. The interest rate (continuously compounding) is r. The forward price, F, of the asset at time t is given by the following equation:

F = S * exp((r-d)*t)

The current values of S, d, r and t are as shown below:

Spot	[$]	100
r	[%]	4.30%
d	[%[	1.20%
t	[yrs]	1.5

Find the sensitivity of the forward price to S, r, d and t.

	Spot	r	d	t	Forward price
Original values:	100	4.30%	1.20%	1.5	104.7598
Increase original spot by 1%:	101	4.30%	1.20%	1.5	105.8074
Add 1% to original r:	100	5.30%	1.20%	1.5	106.3430
Add 1% to original d:	100	4.30%	2.20%	1.5	103.2001
Add 0.01 to original t:	100	4.30%	1.20%	1.51	104.7923

Sensitivity to spot:	[#]	1.047598
Sensitivity to r:	[1/%]	158.3242
Sensitivity to d:	[1/%]	-155.967
Sensitivity to t:	[1/yrs]	3.248057

3 Calculating sensitivity of annuity to its drivers

An investment generates a dividend that grows by g% per year in perpetuity. The first dividend is D and is received in one year's time. Future dividends are discounted back to present value at a discount rate of r. The present value, P, of the dividends generated by the investment is given by the formula P = D/(r-g)

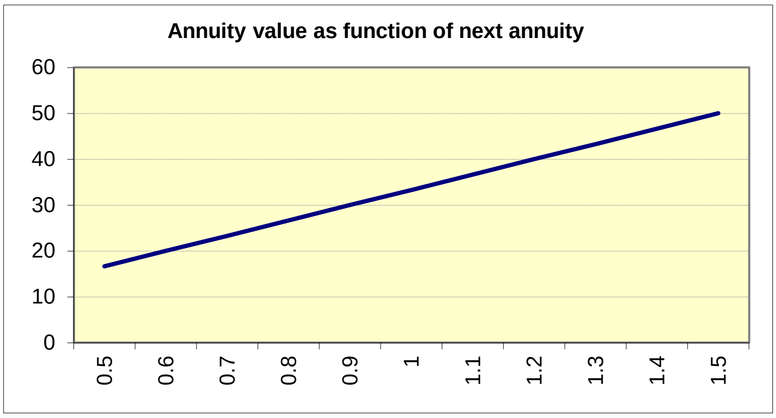
The current values of D, r and g are as shown below:

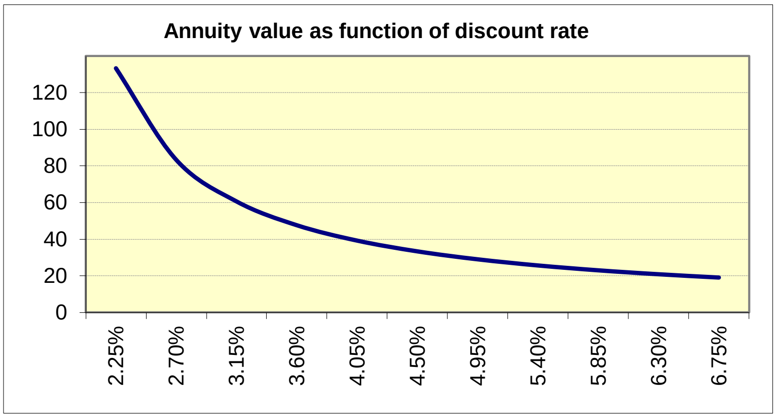
D	1
r	4.5%
g	1.5%

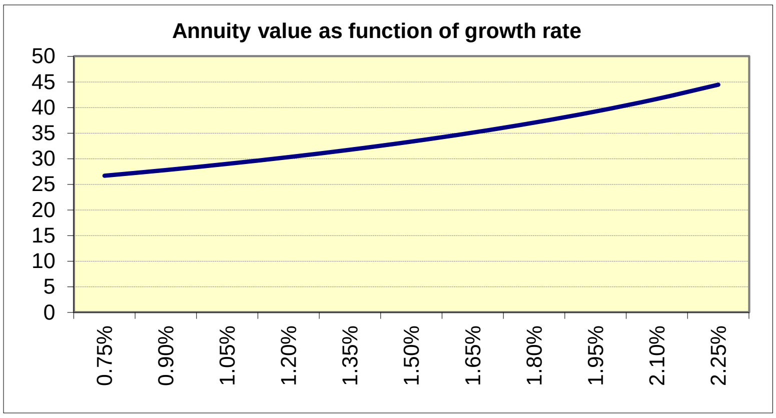
Find the sensitivity of the present value, P, to D, r and g.

	D	r	g	P
Original values:	1	4.50%	1.50%	33.33
Increase D by 1%	1.01	4.50%	1.50%	33.67
Add 1% to original r	1	5.50%	1.50%	25.00
Add 1% to original g	1	4.50%	2.50%	50.00

Sensitivity to D	[#]	33.33333
Sensitivity to r	[1/%]	-833.3333
Sensitivity to g	[1/%]	1666.667







	G	H	I	J	K	L	M	N	O	P	Q	R
134												
147							www.tykoh.com					

Click here to see what your answer should look like.

Gradient is a similar concept to sensitivity. If Y is a function of X then the gradient of Y is the change in Y divided by the change in X.

1 Gradient of e^(ax)

Some common financial functions have gradients that can be expressed as formulae. In this and following questions we look at some of those functions.

Calculate the gradient of e^{2x} when x is 1.5. Show that the gradient is equal to $2.e^{2x}$. We can conclude that, in general, the gradient of e^{ax} is $a.e^{ax}$.

x	e^{2x}	gradient	$2.e^{2x}$
1.5	20.08554	40.171	40.171
1.50001	20.08594		

Gradient is =(G15-G14)/(F15-F14)

2 Gradient of x^n

Calculate the gradient of x^3 when x is 2. Show that the gradient is equal to $3.x^2$. We can conclude that, in general, the gradient of x^n is $n.x^{(n-1)}$.

x	x^3	gradient	$3.x^2$
2	8	12.00	12.00
2.00001	8.00012		

3 Gradient of a^x

Calculate the gradient of 2^x when x is 3. Show that the gradient is equal to $\ln(2).2^x$. We can conclude that, in general, the gradient of a^x is $\ln(a).a^x$.

x	2^x	gradient	$\ln(2).2^x$
3	8	5.55	5.55
3.00001	8.000055		

4 Gradient of ln(x)

Calculating the gradient of ln(x) when x is 2. Show the gradient is equal to 1/2. We can conclude that, in general, the gradient of ln(x) is 1/x.

x	ln(x)	gradient	1/x
2	0.693147	0.50	0.50
2.00001	0.693152		

5 Calculating gradient of the product of two functions

Suppose f and g are functions of x. Then the gradient of f x g is given by the following formula;

Gradient of f(x).g(x) = f ' (x).g(x) + f(x).g ' (x), where ..

f ' (x) is the gradient of f(x) and g ' (x) is the gradient of g(x)

Calculate the gradient of $2^x.x^3$ when x is 2. Show that the gradient is the same as would be predicted by using the rule above. The rule predicts the gradient should be $2^x.x^2(3+\ln(2).x)$

x	$2^x.x^3$	gradient	$x^2.2^x(3+\ln(2).x)$
2	32	70.18	70.18
2.00001	32.0007		

6 Calculating the gradient of a function of a function

g(x) is a function of x. f is a function of g(X). Then the gradient of f(g(x)) is given by a "chain rule":

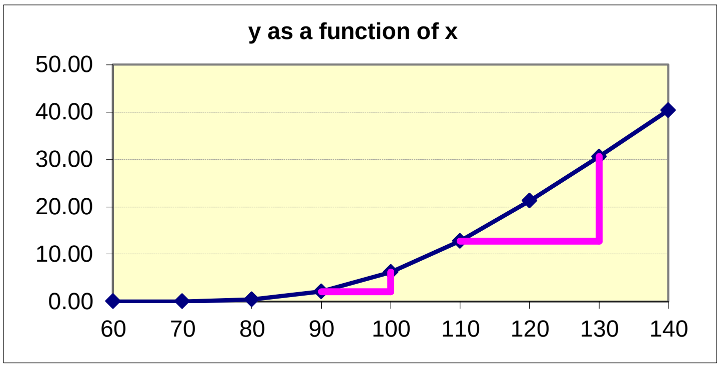
Gradient of f(g(x)) = f ' (g(x)).g ' (x)

Calculate the gradient of $3^{\ln(x)}$ when x is 2. Show that the gradient is the same as would be predicted by using the chain rule. The rule predicts the gradient should be $3^{\ln(x)}.\ln(3)/x$

x	$3^{\ln(x)}$	gradient	$3^{\ln(x)}.\ln(3)/x$
2	2.141486	1.176	1.176
2.00001	2.141498		

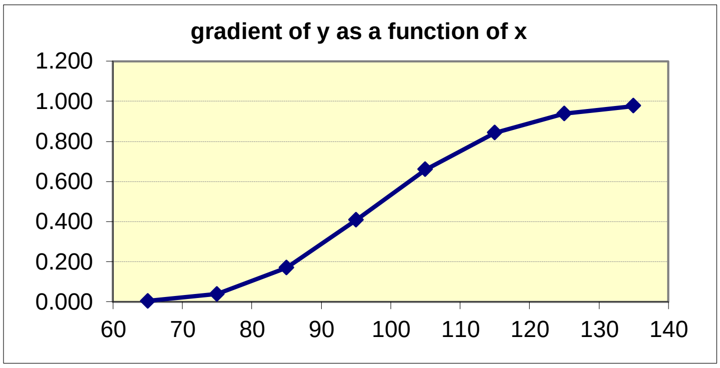
7 Calculating gradient of a piecewise-linear curve.

x	y
60	0.00
70	0.04
80	0.43
90	2.12
100	6.19
110	12.79
120	21.22
130	30.60
140	40.37



The table and chart above show a piecewise-linear "curve". Using that table calculate the gradients at the x values given in the table below.

x	gradient
65	0.004
75	0.039
85	0.169
95	0.407
105	0.660
115	0.843
125	0.938
135	0.977

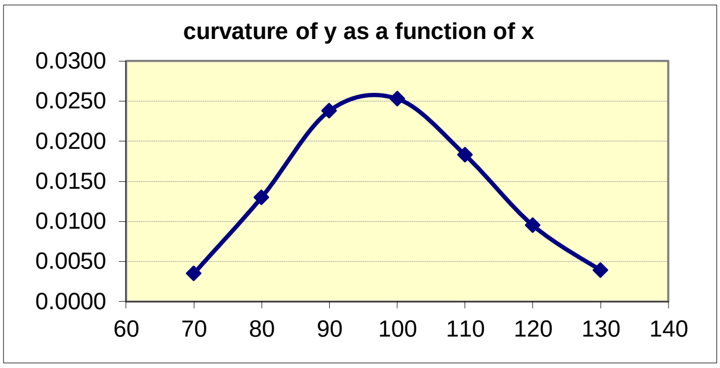


8 Calculating curvature

Curvature is a second-order measure: It is the gradient of the gradient. In other words curvature measures how quickly the gradient of y changes as a function of x.

Continuing with the above example - calculate the curvature of y(x) in the table below.

x	Curvature
70	0.0035
80	0.0130
90	0.0238
100	0.0253
110	0.0183
120	0.0095
130	0.0039



9 Using gradient and curvature in interpolating and extrapolating

Gradient and curvature can be used in interpolating and extrapolating. Consider a function V(x). Suppose the value of the function V(x), its gradient V'(x) and curvature V''(x) are all known at a particular value of x, say x_1. Then the value of V at a slightly different value of x - say x_1+dx is given approximately by the following formula: $V(x_1+dx) = V(x) + V'(x).dx + 0.5\ V''(x).dx^2$

Calculate the approximate value of V(x) when x is 2.1 by using the value of V(x) and its gradient and curvature at x = 2.0

	value	gradient	curvature
x	V(x)	V'(x)	V''(x)
2.0	2.225541	0.890216	0.356087
2.1	2.316367		

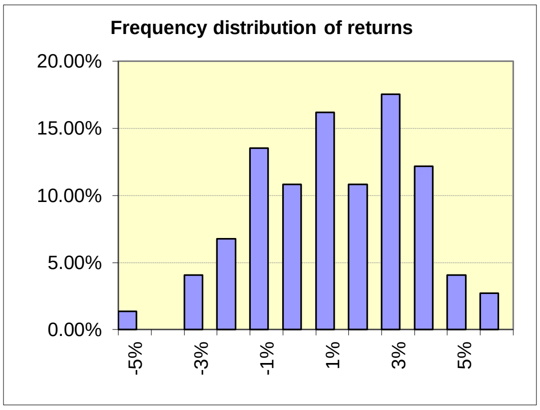
Click here to see what your answer should look like.

1 Calculating a frequency distribution from a sample set

This question and the following ones relate to frequency distributions, mean and mode. All of these are ways of reducing large data sets to represent them in a more concise but still useful form.

The table below shows the number of times a particular security has yielded a given return. Calculate the frequency distribution of the returns. The frequency distribtion is the list of the percentage of times each return has occurred.

Return	Number of times	Frequency [%]
-5%	1	1.35%
-4%	0	0.00%
-3%	3	4.05%
-2%	5	6.76%
-1%	10	13.51%
0%	8	10.81%
1%	12	16.22%
2%	8	10.81%
3%	13	17.57%
4%	9	12.16%
5%	3	4.05%
6%	2	2.70%



2 Calculating a mean from a frequency distribution.

Calculate the mean return.

Mean	1.30%

3 Calculating a mode from a frequency distribution.

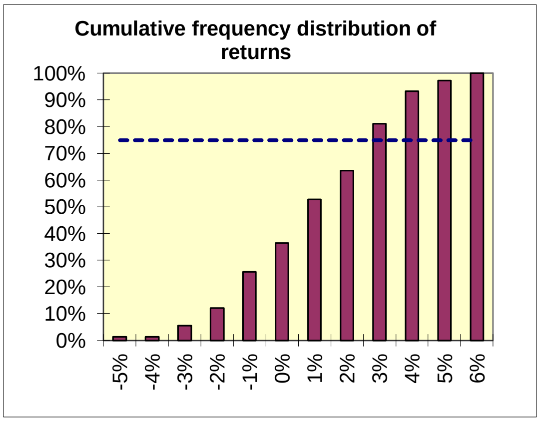
Calculate the modal return (the return that occurred the most often)

Number of times most frequent return has occurred:	13
Corresponding return:	3%

4 Calculating a cumulative frequency distribution.

In cells O49:O60 show the number of times returns have been less than or equal to the return specified. For example, the number in cell O51 should show the number of times returns were less than or equal to -3%. In cells P49:P60 show the cumulative frequency as a percentage.

Return	Number of times	Cumulative frequency [number]	Cumulative frequency [%]
-5%	1	1	1%
-4%	0	1	1%
-3%	3	4	5%
-2%	5	9	12%
-1%	10	19	26%
0%	8	27	36%
1%	12	39	53%
2%	8	47	64%
3%	13	60	81%
4%	9	69	93%
5%	3	72	97%
6%	2	74	100%



5 Calculating lowest return in upper quartile.

What is the lowest return in the upper quartile?

Lowest return in upper quartile:	3.00%

6 Calculating standard deviation.

Both standard deviation and variance are measures of the "spread" of a distribution. Standard deviation is the square root of variance. Both standard deviation and variance can be calculated on "sample" or "population" terms. "Population" means that the data you're working with comprises all the data, "Sample" means that there is more data and you're working with the sampled subset.

Calculate the sample standard deviation of the frequency distribution described in cells M81:N92 below.

Return	Number of times	Number of times x return	Return - mean return	(Return - mean return)^2	Number of times x (Return - mean return)^2
-5%	1	-0.05	-6%	0.003966	0.003966
-4%	0	0	-5%	0.002806	0.000000
-3%	3	-0.09	-4%	0.001847	0.005540
-2%	5	-0.1	-3%	0.001087	0.005436
-1%	10	-0.1	-2%	0.000528	0.005278
0%	8	0	-1%	0.000168	0.001346
1%	12	0.12	0%	0.000009	0.000106
2%	8	0.16	1%	0.000049	0.000395
3%	13	0.39	2%	0.000290	0.003769
4%	9	0.36	3%	0.000730	0.006574
5%	3	0.15	4%	0.001371	0.004113
6%	2	0.12	5%	0.002212	0.004423

Total	74				0.040946

Mean return	1.30%

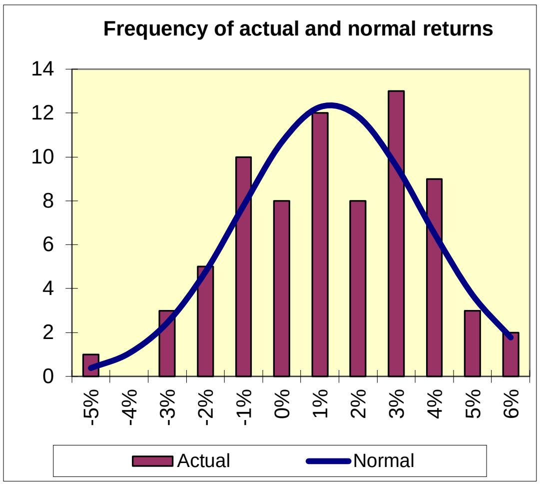
Standard deviation = Square root of [total in column R / (total in column N - 1)]	2.368%

7 Generating a normal distribution

We can generate a normal distribution by using the NORMDIST function. In the question below we use the NORMDIST function to superimpose a normal curve on our actual frequency distribution. This shows visually the extent to which the data set is "Normal".

Cells N114:N127 calculate the cumulative normal distribution using the mean and standard deviation we calculated in cells O97 and T99 respectively. In cells O115:O126 calculate the increments in the cumulative distribution. Scale these by the total number of samples in cell N95 to give the predicted number of normal samples in cells P115:P126.

Return	Cumulative normal dist	Change in Cumulative normal dist	Frequency
-6%	0.0021		
-5%	0.0072	0.0051	0.380
-4%	0.0214	0.0142	1.052
-3%	0.0544	0.0330	2.444
-2%	0.1188	0.0643	4.762
-1%	0.2240	0.1052	7.784
0%	0.3682	0.1442	10.673
1%	0.5341	0.1659	12.278
2%	0.6942	0.1601	11.848
3%	0.8238	0.1296	9.592
4%	0.9119	0.0880	6.514
5%	0.9620	0.0502	3.711
6%	0.9860	0.0240	1.774
7%	0.9956		



8 Calculating percentage of outcomes less than N standard deviations above mean

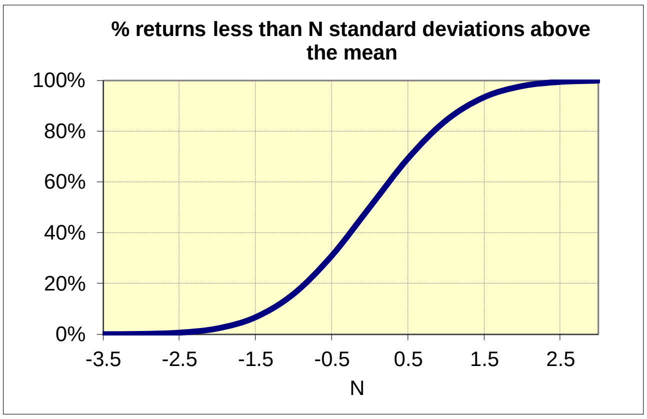
We can use the NORMSDIST function to give the proportion of values less than a given number of standard deviations above the mean.

An investment has normal returns. What percentage of returns will be less than one standard deviation (σ) above the mean?

% returns < (mean + σ):	84.1%

Complete the table below showing the percentage of returns less than N standard deviations above the mean.

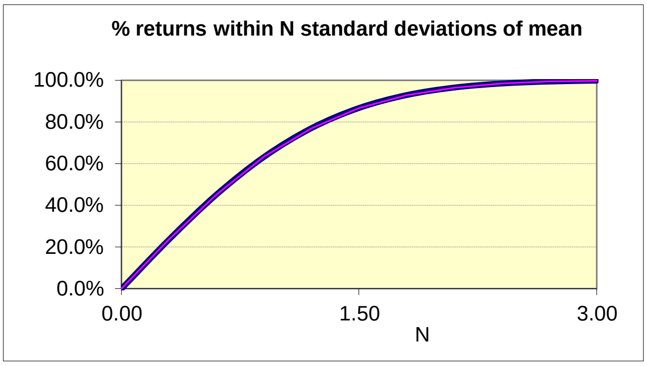
N	% returns
-3.5	0.02%
-3.0	0.13%
-2.5	0.62%
-2.0	2.28%
-1.5	6.68%
-1.0	15.87%
-0.5	30.85%
0.0	50.00%
0.5	69.15%
1.0	84.13%
1.5	93.32%
2.0	97.72%
2.5	99.38%
3.0	99.87%



9 Calculating percentage of outcomes within N standard deviations of the mean

Complete the table below showing the percentage of returns within N standard deviations from the mean.

N	Percentage
0.00	0.0%
0.30	23.6%
0.60	45.1%
0.90	63.2%
1.20	77.0%
1.50	86.6%
1.80	92.8%
2.10	96.4%
2.40	98.4%
2.70	99.3%
3.00	99.7%



10 Calculating number of standard deviations above mean below which a given percentage of samples occur.

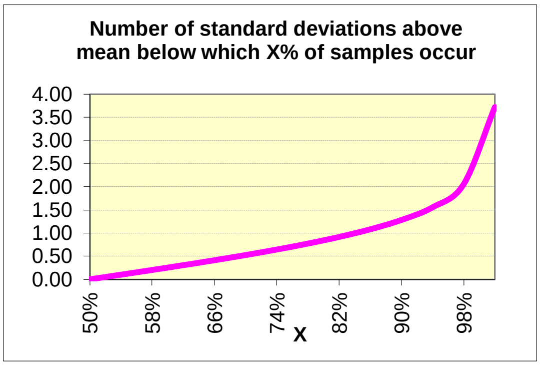
We can use the NORNMSINV function to give the number of standard deviations above the mean below which a certain proportion of outcomes will occur.

85% of returns occur less than N standard deviations above the mean. What is N?

N	1.036433

Complete the following table which shows the number of standard deviations above the mean below which X% of samples occur.

X	Number Sd's
50.00%	0.00000
54.00%	0.10043
58.00%	0.20189
62.00%	0.30548
66.00%	0.41246
70.00%	0.52440
74.00%	0.64335
78.00%	0.77219
82.00%	0.91537
86.00%	1.08032
90.00%	1.28155
94.00%	1.55477
98.00%	2.05375
99.99%	3.71902



11 Generating samples from a normal distribution.

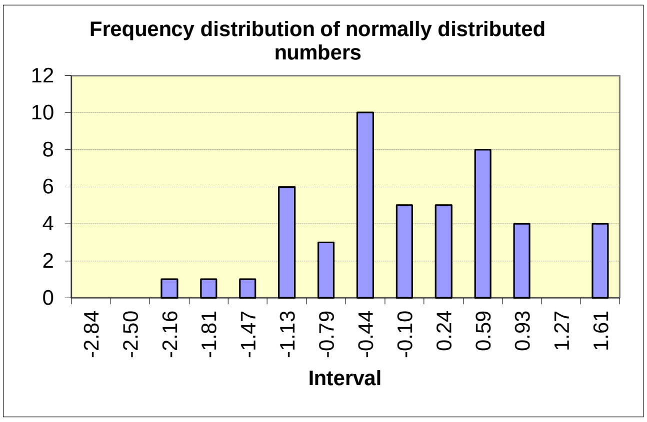
The RAND function can be used to generate samples drawn from a rectangular distribution. The distribution can be "mapped" to another distribution. For example, to generate samples drawn from a normal distribution you can pass the numbers generated by RAND to the NORMSINV function. The NORMSINV function will then generate samples drawn from a normal distribution that has a mean of zero and standard deviation of 1.

The samples below have been generated by using the =RAND() function. That function generates numbers that are equally likely to fall anywhere in the range from 0 to 1. In other words, the samples have a "rectangular" distribution.

M	N	O	P	Q	R	S	T	U	V
0.2216	0.3150	0.5566	0.6857	0.7798	0.6219	0.2636	0.6221	0.0389	0.2515
0.3908	0.3954	0.0936	0.6733	0.9432	0.2704	0.5687	0.5744	0.4195	0.1504
0.1079	0.1321	0.1018	0.0828	0.2485	0.4235	0.9109	0.2288	0.9881	0.4921
0.0238	0.2703	0.7686	0.1145	0.1044	0.0064	0.6685	0.9453	0.7188	0.2208
0.5004	0.4223	0.6263	0.3240	0.9859	0.6059	0.7956	0.7289	0.9280	0.2045

	L	M	N	O	P	Q	R	S	T	U	V	W
230												
231		Use the numbers above to generate normally distributed numbers.										
232												
233		-0.7667	-0.4816	0.1424	0.4837	0.7714	0.3104	-0.6323	0.3110	-1.7642	-0.6698	
234		-0.2771	-0.2653	-1.3189	0.4491	1.5824	-0.6115	0.1731	0.1876	-0.2032	-1.0345	
235		-1.2379	-1.1167	-1.2713	-1.3865	-0.6792	-0.1929	1.3463	-0.7429	2.2601	-0.0199	
236		-1.9814	-0.6118	0.7341	-1.2032	-1.2571	-2.4880	0.4359	1.6011	0.5791	-0.7694	
237		0.0010	-0.1960	0.3220	-0.4567	2.1952	0.2686	0.8259	0.6093	1.4614	-0.8256	
238												
239		Chart the frequency distribution of the normally distributed numbers. Use the FREQUENCY function.										
240												
241		Min number:		-2.4880		Rounded min:		-2.5000		Step size:	0.343	
242		Max number:		2.2601		Rounded max:		2.3000				
243												

	Interval	Number
246	-2.84	0
247	-2.50	0
248	-2.16	1
249	-1.81	1
250	-1.47	1
251	-1.13	6
252	-0.79	3
253	-0.44	10
254	-0.10	5
255	0.24	5
256	0.59	8
257	0.93	4
258	1.27	0
259	1.61	4



261
262
263
264

Click here to see what your answer should look like.

1 Calculating sample variance

Variance is the square of the standard deviation. One method of calculating standard deviation is shown in an earlier tab in this topic. Another way - which can be used if you have individual samples - is to use the spreadsheet function STDEV.

Calculate the standard deviation and variance of the following set of samples:

0%	-2%	-5%	-1%	-3%	-2%	1%	1%
0%	3%	4%	-2%	3%	2%	-1%	2%
5%	4%	3%	-3%	-1%	0%	-1%	4%
1%	-2%	5%	-2%	0%	1%	1%	0%
2%	-1%	-1%	-1%	0%	-1%	6%	3%
3%	4%	3%	4%	-1%	-3%	3%	4%
1%	3%	1%	0%	1%	6%	3%	2%
1%	2%	1%	0%	5%	2%	4%	1%
2%	3%	4%	3%	2%	3%	4%	3%
-1%	1%						

Standard deviation:	[%]	2.368%
Variance (square of cell J21).	[%]	0.000561
Variance (using VAR function)		0.000561

2 Calculating population variance

If your data encompasses the entire population you should use the STDEVP or VARP functions to calculate the population standard deviation and variance respectively. Assume the data above comprises the entire population. Calculate the population standard deviation and variance.

Standard deviation:	[%]	2.352%
Variance (square of cell J31).	[%]	0.000553
Variance (using VARP function)		0.000553

3 Calculating population covariance

Consider the data for three series (X,Y & Z) below. Calculate the (population) covariances between the series by using the COVAR function.

X	Y	Z
1.2	3	5
1.7	3.2	4.9
2.1	3.1	5.5
2.2	3.4	7

	Covariance matrix		
	X	Y	Z
X	0.1550	0.0425	0.2400
Y	0.0425	0.0219	0.1025
Z	0.2400	0.1025	0.7050

Determine the (population) standard deviations of the X, Y and Z series.

Standard deviations		
X	Y	Z
0.3937	0.1479	0.8396

4 Calculating population correlation

Determine the correlations between the X, Y and Z series by using the CORREL function.

	Correlation matrix		
	X	Y	Z
X	1.0000	0.7299	0.7260
Y	0.7299	1.0000	0.8254
Z	0.7260	0.8254	1.0000

5 Calculating correlation matrix from covariance matrix

The correlation between x and y - correl(x,y) - is equal to covar(x,y)/(sd(x)*sd(y)) where covar(x,y) is the covariance between x and y, and sd(x) and sd(y) are the standard deviations of x and y respectively.

Calculate the correlation matrix below by using the covariance and standard deviation matrices above.

	Correlation matrix		
	X	Y	Z
X	1.0000	0.7299	0.7260
Y	0.7299	1.0000	0.8254
Z	0.7260	0.8254	1.0000

6 Calculating covariance matrix from correlation matrix and standard deviation

The covariance between x and y - covar(x,y) - is equal to correl(x,y)*sd(x)*sd(y) where correl(x,y) is the correlation between x and y, and sd(x) and sd(y) are the standard deviations of x and y respectively.

Calculate the covariance matrix below by using the correlation and standard deviation matrices above.

	Covariance matrix		
	X	Y	Z
X	0.1550	0.0425	0.2400
Y	0.0425	0.0219	0.1025
Z	0.2400	0.1025	0.7050

7 Calculating volatility

Volatility is the standard deviation of the "log-returns" of an asset. The "log-return" r is given by r = $\ln(a_{t+1}/a_t)$ where a_{t+1} is the price of the asset at time t+1 and a_t is the price at time t.

Calculate the volatility of the asset whose daily prices are shown below.

Day	Price	Log return
1	100.0	
2	100.7	0.70%
3	101.2	0.50%
4	101.1	-0.10%
5	100.5	-0.60%
6	99.9	-0.60%
7	99.9	0.00%
8	98.7	-1.21%
9	99.5	0.81%
10	99.1	-0.40%
11	97.9	-1.22%
12	98.8	0.92%
13	100.1	1.31%
14	99.4	-0.70%
15	100.4	1.00%
16	101.2	0.79%
17	102.4	1.18%
18	103.1	0.68%
19	103.4	0.29%

Standard deviation of log return: [%] 0.81% <- Use STDEV

The volatility above is the daily volatility. To convert to a yearly volatility multiply by the square root of the number of trading days in a year.

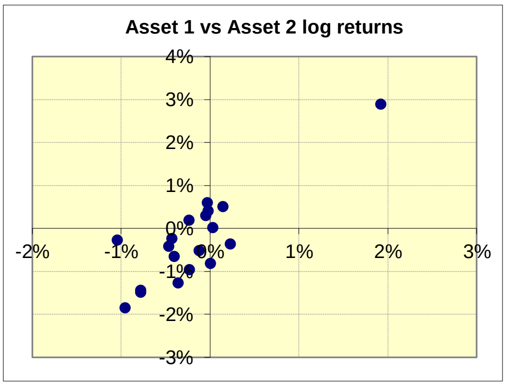
Number of trading days in year: [#] 252

Yearly volatility [%] 12.89%

8 Calculating correlation of log returns

Calculate the correlation between the log returns of the two assets below.

Day	Asset 1 price	Asset 2 price	Asset 1 log return	Asset 2 log return
1	1.1789	1.9100		
2	1.1790	1.8944	0.01%	-0.82%
3	1.1743	1.8820	-0.40%	-0.66%
4	1.1632	1.8474	-0.95%	-1.86%
5	1.1659	1.8406	0.23%	-0.37%
6	1.1569	1.8134	-0.77%	-1.49%
7	1.1520	1.8090	-0.42%	-0.24%
8	1.1744	1.8620	1.93%	2.89%
9	1.1653	1.8353	-0.78%	-1.44%
10	1.1626	1.8388	-0.23%	0.19%
11	1.1612	1.8292	-0.12%	-0.52%
12	1.1616	1.8295	0.03%	0.02%
13	1.1575	1.8064	-0.35%	-1.27%
14	1.1572	1.8171	-0.03%	0.59%
15	1.1567	1.8225	-0.04%	0.30%
16	1.1584	1.8317	0.15%	0.50%
17	1.1582	1.8391	-0.02%	0.40%
18	1.1529	1.8313	-0.46%	-0.43%
19	1.1410	1.8262	-1.04%	-0.28%
20	1.1384	1.8087	-0.23%	-0.96%



Correlation	[%]	84.92%

Click here to see what your answer should look like.

Interpolation involves "filling in the gaps" between discrete data points. In this section we review and apply various interpolation methods including two of the most used: Linear and cubic spline interpolation. We also look at "curve fitting" and at extrapolation.

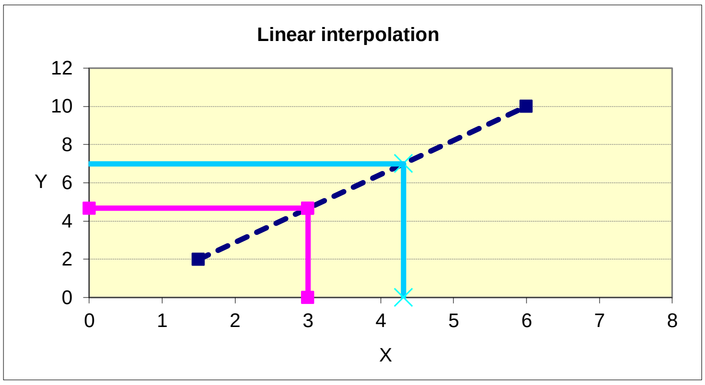
1 Linear interpolation

Consider the two (x,y) data points defined in cells J12:J15. Calculate by using linear interpolation the y value that corresponds to the x value in cell J17. Also calculate the x value that correponds to the y value in cell J20.

x1	1.5
x2	6
y1	2
y2	10

x value to interpolate	3
y at that x	4.667

y value to interpolate	7
x at that y	4.313



2 Cubic spline interpolation

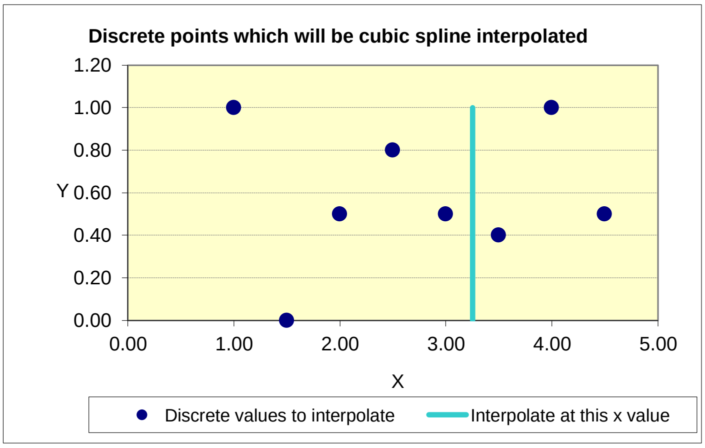
Cubic spline interpolation fits a curve (cubic) between data points. In the method described below the leftmost and rightmost curve segments have zero curvature at the leftmost and rightmost points respectively.

[For more information on this method see: http://www.scribd.com/doc/24255441/Cubic-Spline-Interpolation

Consider the table of (x,y) values shown below in cells J40:K47. Use cubic spline interpolation to find the y value corresponding to the x value in cell K49.

Table to interpolate		
i (interval)	x_i	y_i
1	1.00	1.00
2	1.50	0.00
3	2.00	0.50
4	2.50	0.80
5	3.00	0.50
6	3.50	0.40
7	4.00	1.00
8	4.50	0.50

x value to interpolate	3.25



Step 1 - Calculate "hh" - the interval between successive x values. [x values are spaced evenly - so you can calculate the difference any two successive intrervals.]

hh	0.5

Step 2 - Calculate which interval the interpolated x value is in

i	5	<- =FLOOR((x_to_interpolate-first_x)/hh,1)+1

Step 3 - Calculate x's offset within the interval it is in

dX	0.25	<- =x_to_interpolate-first_x-(i-1)*hh

Step 4 - Calculate a vector of curvatures of y_i

i	y_i	$Curvature_i$	
1	1		
2	0	36	<- Copy and paste this formula down
3	0.5	-4.8	
4	0.8	-14.4	
5	0.5	4.8	
6	0.4	16.8	
7	1	-26.4	
8	0.5		

Step 5 - Generate an "M" matrix.

i	"M" matrix						
1							
2	4	1	0	0	0	0	
3	1	4	1	0	0	0	
4	0	1	4	1	0	0	
5	0	0	1	4	1	0	<- Continue the pattern on the
6	0	0	0	1	4	1	two rows below.
7	0	0	0	0	1	4	
8							

Step 6 - Generate an 'M' vector in cells K97:K102 by multiplying the inverse of the 'M' matrix in J84:O89 by the y curvatures in cells K71:K76.

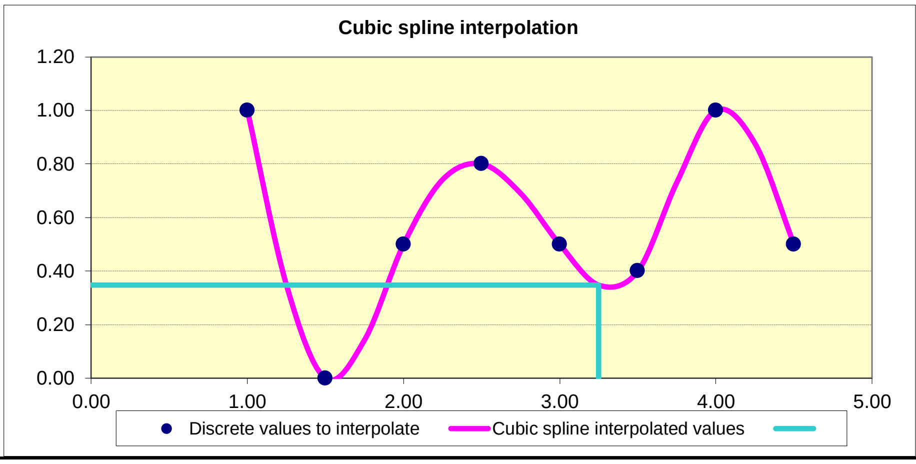
i	y_i	M_i
1	1	0
2	0	9.721195
3	0.5	-2.88478
4	0.8	-2.98207
5	0.5	0.413054
6	0.4	6.129852
7	1	-8.13246
8	0.5	0

Step 7 - Calculate the interpolated y values within each interval (in the step following we will choose only one of the intervals)

i	x_i	y_i	M_i	a_i	b_i	c_i	y	
1	1.0	1	0	3.240398	0	-2.8101	0.348106	<- Copy and paste
2	1.5	0	9.721195	-4.20199	4.860598	-0.3798	0.143181	these formulae down
3	2.0	0.5	-2.88478	-0.03243	-1.44239	1.329303	0.74167	
4	2.5	0.8	-2.98207	1.131707	-1.49103	-0.13741	0.690141	
5	3.0	0.5	0.413054	1.905599	0.206527	-0.77966	0.347767	
6	3.5	0.4	6.129852	-4.75411	3.064926	0.856063	0.731291	
7	4.0	1	-8.13246	2.710821	-4.06623	0.355411	0.87707	
8	4.5	0.5	0					

Step 8 - Choose the interpolated y value in the interval we calculated in step 2.

Interpolated y value: 0.347767

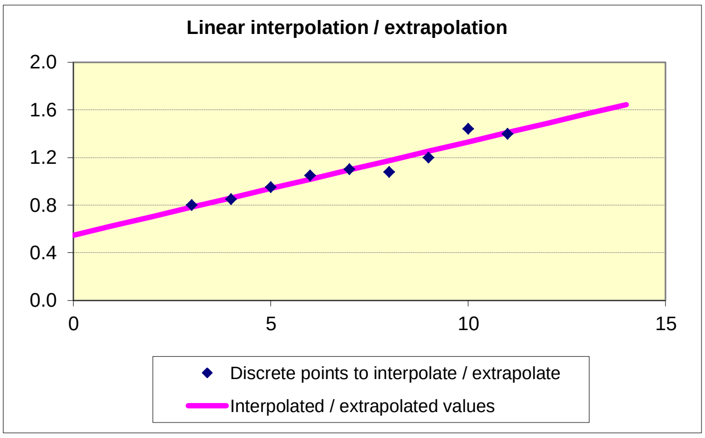


3 TREND function to perform linear interpolation / extrapolation

The TREND function performs a "least-squares" straight-line fit to a set of data points. Another such function is the GROWTH function - that function performs a fit to an exponential or growth curve. Other types of curve can be fitted to by transforming them into straight lines, using TREND and then reversing the transformation. Examples of these techniques are covered in this and the following questions.

Consider the table of (x,y) values shown below in cells I157:J165. Use the TREND function to perform a straight-line fit to those points and to calculate the y values corresponding to the x values in cells I154 to I168.

x	y	Interpolate
0		0.5483
1		0.6267
2		0.7050
3	0.80	0.7833
4	0.85	0.8617
5	0.95	0.9400
6	1.05	1.0183
7	1.10	1.0967
8	1.08	1.1750
9	1.20	1.2533
10	1.44	1.3317
11	1.40	1.4100
12		1.4883
13		1.5667
14		1.6450

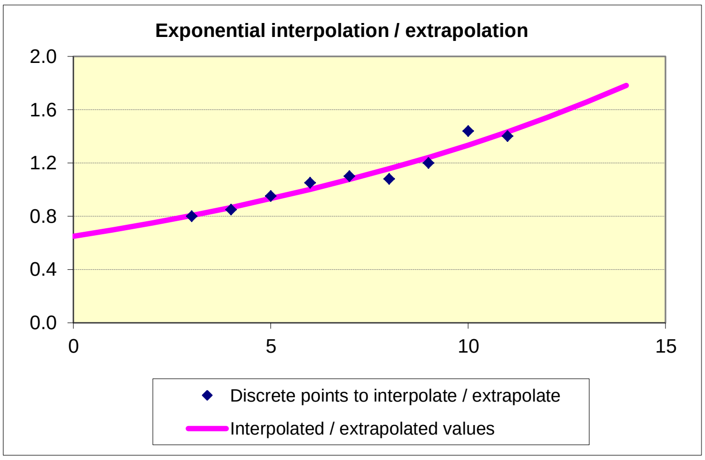


4 GROWTH function to perform linear interpolation / extrapolation

The GROWTH function assumes the data points to be fitted are on an exponentially growing curve. In this question we use the same data points as in the preceding question but now we assume the underlying function is exponential rather than linear.

Use the GROWTH function to fit an exponential curve to the (x,y) data points listed below.

x	y	Interpolate
0		0.6510
1		0.6995
2		0.7517
3	0.80	0.8077
4	0.85	0.8680
5	0.95	0.9327
6	1.05	1.0023
7	1.10	1.0770
8	1.08	1.1573
9	1.20	1.2436
10	1.44	1.3363
11	1.40	1.4360
12		1.5431
13		1.6582
14		1.7818

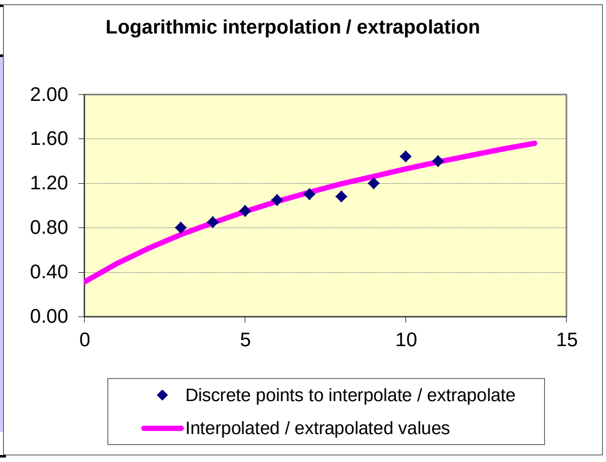


5 Fitting to a logarithmic curve

You can fit points to an arbitrary underlying function by first transforming the function to a straight line, using the straight line interpolation function TREND and then reversing the original transformation. The questions below illustrate that technique.

Fit a natural log curve to the (x,y) data points defined in the table below.

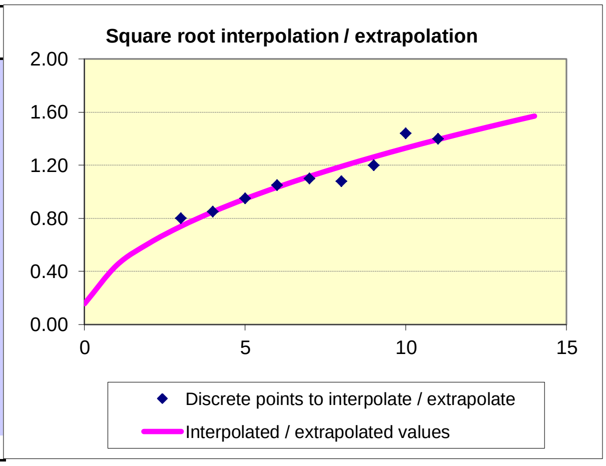
Row	x	y	exp(y)	Interpolate	ln(Interpolate)
206	0			1.3678	0.3132
207	1			1.6097	0.4761
208	2			1.8517	0.6161
209	3	0.80	2.2255	2.0936	0.7389
210	4	0.85	2.3396	2.3356	0.8483
211	5	0.95	2.5857	2.5776	0.9468
212	6	1.05	2.8577	2.8195	1.0366
213	7	1.10	3.0042	3.0615	1.1189
214	8	1.08	2.9447	3.3035	1.1950
215	9	1.20	3.3201	3.5454	1.2657
216	10	1.44	4.2207	3.7874	1.3317
217	11	1.40	4.0552	4.0293	1.3936
218	12			4.2713	1.4519
219	13			4.5133	1.5070
220	14			4.7552	1.5592



6 Fitting to a square-root curve

Fit a square root curve to the (x,y) data points defined in the table below.

Row	x	y	y^2	Interpolate	$(Interpolate)^{1/2}$
228	0			0.0246	0.1569
229	1			0.1992	0.4463
230	2			0.3737	0.6113
231	3	0.80	0.6400	0.5482	0.7404
232	4	0.85	0.7225	0.7228	0.8502
233	5	0.95	0.9025	0.8973	0.9473
234	6	1.05	1.1025	1.0719	1.0353
235	7	1.10	1.2100	1.2464	1.1164
236	8	1.08	1.1664	1.4209	1.1920
237	9	1.20	1.4400	1.5955	1.2631
238	10	1.44	2.0736	1.7700	1.3304
239	11	1.40	1.9600	1.9445	1.3945
240	12			2.1191	1.4557
241	13			2.2936	1.5145
242	14			2.4681	1.5710



Click here to see what your answer should look like.

1 Calculating asset price distribution in one period's time.

Assume that over a certain period an asset's value can change by +10% or by -10% and that each of these possibilities are equally likely. The asset's current price is $100. Calculate the probability distribution of the asset's price in one period's time. The table below should be ranked in decreasing order of price.

Price in one period	Probability	Return
110	50%	10%
90	50%	-10%

Calculate the expected or mean return over the period.

Expected / mean return	0.00%

Calculate the volatility. [Use the STDEVP function.]

Volatility	10%

2 Calculating asset price distribution in two period's time.

With the same asset as in the preceding question - what is its price probability distribution in two period's time?

Price in two weeks	Probability
121	25%
99	50%
81	25%

What is the expected (mean, average) price in two period's time?

Expected price	100.000

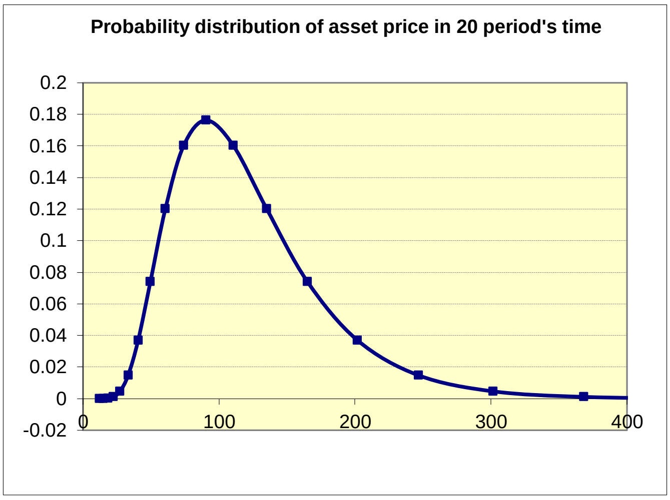
What is the modal (most likely, highest probability) price in two period's time?

Modal price	99

3 Calculating asset price distribution "N" periods into the future.

Consider the same asset as in the preceding question. What is its price distribution in 20 period's time? [Use the BINOMDIST function to calculate the probabilities]

Number of "up" moves in 20 periods	Price in 20 periods	Probability
20	672.74999	9.537E-07
19	550.43181	1.907E-05
18	450.3533	0.0001812
17	368.47088	0.0010872
16	301.47618	0.0046206
15	246.66233	0.0147858
14	201.81463	0.0369644
13	165.12106	0.0739288
12	135.09905	0.1201344
11	110.53559	0.1601791
10	90.438208	0.1761971
9	73.994897	0.1601791
8	60.541279	0.1201344
7	49.533774	0.0739288
6	40.527633	0.0369644
5	33.158973	0.0147858
4	27.130069	0.0046206
3	22.197329	0.0010872
2	18.161451	0.0001812
1	14.859369	1.907E-05
0	12.157665	9.537E-07



What is the expected (mean, average) price in twenty period's time?

Expected price	100.000

What is the modal (most likely, highest probability) price in 20 period's time?

Modal price	90.438208

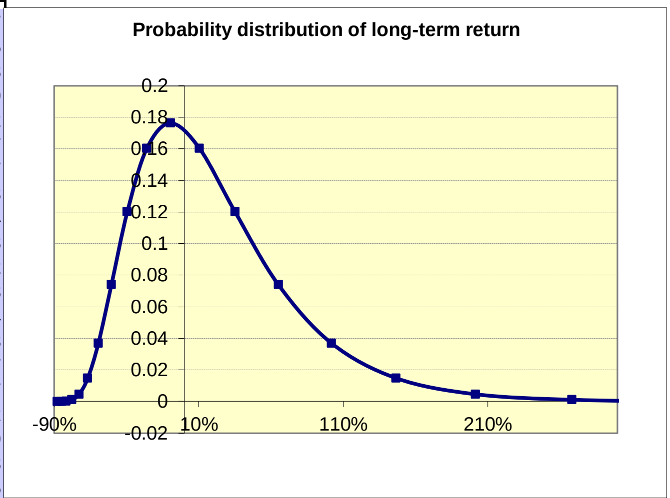
What is the approximate probability that the asset's value in 20 period's time will be less than its current value? [Add up the probabilities in the table above that correspond to an asset price of less than 100.]

Probability:	58.8%

4 Calculating probability distribution of long-term return.

Define the long-term return over "N" periods as being the percentage change in asset price over that interval. Calculate the probability distribution of the long-term return of the asset above.

Number of "up" moves in 20 periods	Price in 20 periods	Long-term return [%]	Probability
20	672.74999	573%	9.53674E-07
19	550.43181	450%	1.90735E-05
18	450.3533	350%	0.000181198
17	368.47088	268%	0.001087189
16	301.47618	201%	0.004620552
15	246.66233	147%	0.014785767
14	201.81463	102%	0.036964417
13	165.12106	65%	0.073928833
12	135.09905	35%	0.120134354
11	110.53559	11%	0.160179138
10	90.438208	-10%	0.176197052
9	73.994897	-26%	0.160179138
8	60.541279	-39%	0.120134354
7	49.533774	-50%	0.073928833
6	40.527633	-59%	0.036964417
5	33.158973	-67%	0.014785767
4	27.130069	-73%	0.004620552
3	22.197329	-78%	0.001087189
2	18.161451	-82%	0.000181198
1	14.859369	-85%	1.90735E-05
0	12.157665	-88%	9.53674E-07



What is the expected long-term return?

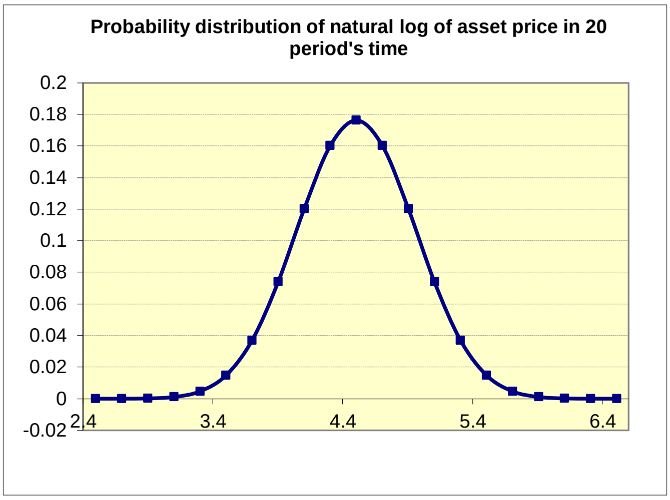
Expected long-term return:	0.00%

5 Calculating probability distribution of logarithm of asset price in "N" period's time.

The distribution above is log-normal (as long as time intervals are relatively short). We can confirm the log-normal nature of the distribution by charting the distribution of the logarithm of the asset price: It should be normal.

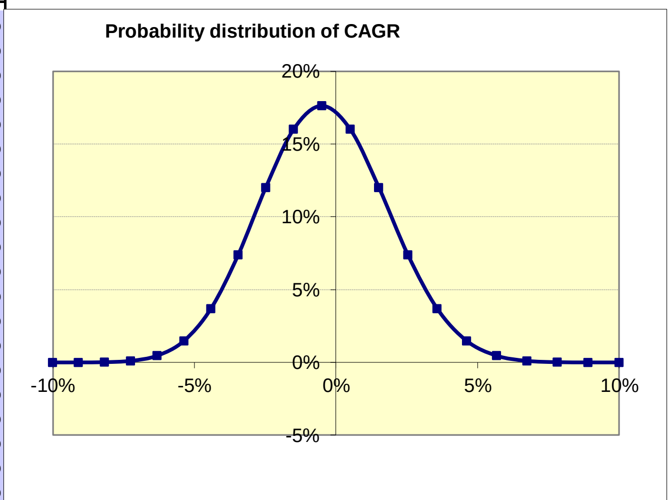
In the table below calculate the natural log of the asset price and the probability of that occuring.

Number of "up" moves in 20 periods	Ln of price in 20 periods	Probability
20	6.5113738	9.537E-07
19	6.3107031	1.907E-05
18	6.1100324	0.0001812
17	5.9093617	0.0010872
16	5.708691	0.0046206
15	5.5080203	0.0147858
14	5.3073496	0.0369644
13	5.1066789	0.0739288
12	4.9060082	0.1201344
11	4.7053375	0.1601791
10	4.5046668	0.1761971
9	4.3039961	0.1601791
8	4.1033254	0.1201344
7	3.9026547	0.0739288
6	3.701984	0.0369644
5	3.5013134	0.0147858
4	3.3006427	0.0046206
3	3.099972	0.0010872
2	2.8993013	0.0001812
1	2.6986306	1.907E-05
0	2.4979599	9.537E-07



6 Calculating distribution of compound average growth rates over "N" periods.

Number of "up" moves in 20 periods	Price in 20 periods	Probability	Per period CAGR [%]
20	672.74999	9.537E-07	10.00%
19	550.43181	1.907E-05	8.90%
18	450.3533	0.0001812	7.81%
17	368.47088	0.0010872	6.74%
16	301.47618	0.0046206	5.67%
15	246.66233	0.0147858	4.62%
14	201.81463	0.0369644	3.57%
13	165.12106	0.0739288	2.54%
12	135.09905	0.1201344	1.52%
11	110.53559	0.1601791	0.50%
10	90.438208	0.1761971	-0.50%
9	73.994897	0.1601791	-1.49%
8	60.541279	0.1201344	-2.48%
7	49.533774	0.0739288	-3.45%
6	40.527633	0.0369644	-4.42%
5	33.158973	0.0147858	-5.37%
4	27.130069	0.0046206	-6.31%
3	22.197329	0.0010872	-7.25%
2	18.161451	0.0001812	-8.18%
1	14.859369	1.907E-05	-9.09%
0	12.157665	9.537E-07	-10.00%



Expected / mean CAGR	-0.48%

7 Matching volatility and growth rate when evolving an asset's price.

Up till now we have assumed the asset's up and down moves in each time step are +/- 10%. If we want to match a particular real-world growth rate and volatility then we need to calculate the up and down moves. There are various ways of doing this. One is illustrated below.

An asset has a yearly return volatility of 30%. Its growth rate is 20% p.a. (continuously compounding). Its current price is 100. What is its expected price in one month's time?

Expected price in one month:	101.680633

Assume that in each month the asset can change in price from S to S*u or to S*d. Each possibility is equally likely. u and d are defined this way:

$$u = e^{g\delta t} + \sigma(\delta t)^{1/2}$$

$$d = e^{g\delta t} - \sigma(\delta t)^{1/2}$$

g is the growth rate, δt is the time interval (one month i.e. 1/12 years) and σ is the volatility.

Calculate the "u" (up) and "d" (down) factors.

u factor	1.1034089
d factor	0.9302038

Determine the asset price in one month if the current asset price (100) is multiplied by the u (up) or d (down) factors.

	Price in one month.	Probability
"up" price	110.34089	50%
"down" price	93.020379	50%

Find the expected price (i.e. the mean or average) in one month's time.

Expected price (mean of "up" and "down" prices):	101.680633

Find the growth rate (p.a.) that is implied by having an asset price now of 100 and an asset price in one month of that shown in cell K208.

Growth rate implied by expected price:	20%

You should find that the implied growth rate in cell K213 matches the asset's assumed growth rate of 20%.

Find the one-month returns implied by the asset prices in cells H203:H204.

One-month return implied by asset's "up" price in one month.	10.34%
One-month return implied by asset's "down" price in one month.	-6.98%

Find the yearly volatility implied by the returns in cells K220:K221.

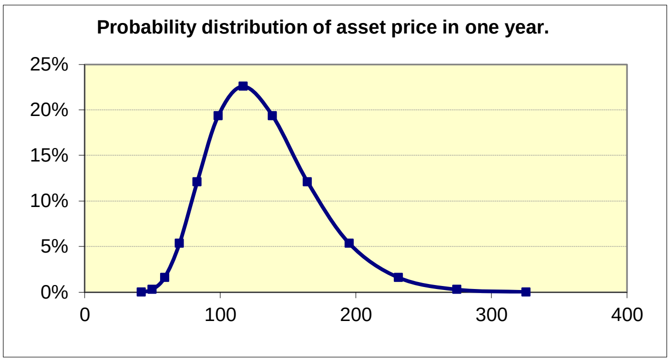
Yearly volatiliy implied by the one month returns in cells K220:K221.	30%

You should find that the volatility in cell K225 matches the asset's known volatility of 30%.

We have confirmed that our "up" and "down" factors generate an expected return and volatility that matches the asset's assumed values.

Calculate the probability distribution of the asset's price in one year.

Number of "up" moves in one year.	Price in one year.	Probability
12	325.71492	0.0002441
11	274.58657	0.0029297
10	231.48397	0.0161133
9	195.1473	0.0537109
8	164.5145	0.1208496
7	138.69021	0.1933594
6	116.91964	0.2255859
5	98.566444	0.1933594
4	83.094202	0.1208496
3	70.05068	0.0537109
2	59.054635	0.0161133
1	49.784669	0.0029297
0	41.969834	0.0002441



8 Calculating asset price distribution by using continuous log-normal distribution.

We can save ourselves some work in calculating the future asset price if we know or assume that short-term asset returns are normally distributed (as they have been on this worksheet). In this case the log of the future asset price is normally distributed. The mean and standard deviation of the natural logarithm of the future asset price is given by the following formulae:

$$\mu = \ln(S) + (g-\sigma^2/2)T$$
$$sd = \sigma * T^{1/2}$$

where μ is the mean of the log distribution, sd is its standard deviation, g is the continuously compounding short-term growth rate of the asset, σ is the asset's yearly return volatility and T is the time (in years) at which we are calculating the asset price distribution.

We'll calculate the distribution of asset prices for the case we considered above.

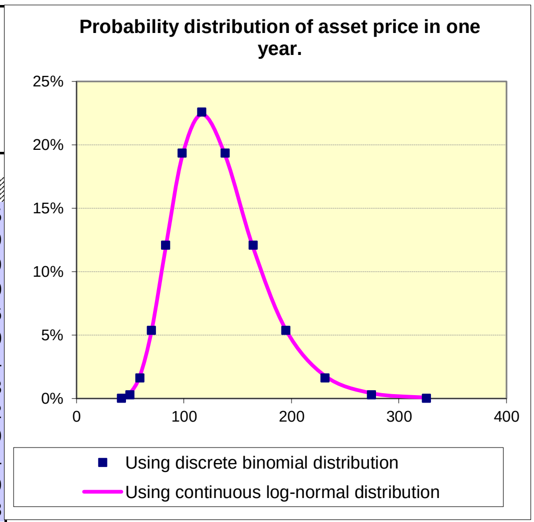
Calculate the mean μ and standard deviation sd of the log of future asset prices using the two formulae above.

Mean	4.7602
sd	30%

In the table below the asset price column contains asset prices that are "evenly" spaced: Each asset price is a fixed multiple of the asset price below it. [In this case the "even" spacing is in a geometrical rather than arithmetic sense.]

i) In cells G289:G302 calculate the natural logs of the asset prices that are in the column to the left.
ii) Cells H289:H302 are the same as the cells to the left except they are 'shifted' by half a division. So, for example, cell H289 contains a number that is midway between the numbers in cells G289 and G290.
iii) In cells I289:I302 calculate the probability that the log of the asset price in one year's time will be less than or equal to the log of the asset price in column H. To do this you will need to use a function that calculates cumulative normal distributions.
iv) In cells J289:J302 calculate the difference between successive cumulative normal probabilities. This will determine the probability that a future asset price will fall into the corresponding interval.

Asset price	ln(Asset price)	Shifted by 1/2 division	Cumulative normal probability	Difference between successive cumulative normal probabilities
386.36	5.95678	5.87140	0.99989	
325.71	5.78602	5.70064	0.99914	0.00075
274.59	5.61527	5.52989	0.99485	0.00429
231.48	5.44451	5.35913	0.97706	0.01779
195.15	5.27375	5.18838	0.92326	0.05380
164.51	5.10300	5.01762	0.80460	0.11866
138.69	4.93224	4.84686	0.61370	0.19090
116.92	4.76149	4.67611	0.38966	0.22404
98.57	4.59073	4.50535	0.19783	0.19183
83.09	4.41997	4.33460	0.07801	0.11982
70.05	4.24922	4.16384	0.02342	0.05459
59.05	4.07846	3.99309	0.00528	0.01814
49.78	3.90771	3.82233	0.00089	0.00439
41.97	3.73695	3.65157	0.00011	0.00078



9 Calculating confidence intervals on an asset's future price.

An asset has a spot price of $80, an expected yearly return of 8%, and a volatility of 25%. Within a confidence level of 99.0% - what is the lowest value the asset's price be in 0.5 years.?

Spot price [S]	[$]	80
Expected return [μ]	[%]	8%
Volatility [σ]	[%]	25%
Future time [T]	[yrs]	0.5
Confidence level [p]	[%]	99.0%

Mean of $\ln(S_T) = \ln(S) + (\mu - \sigma^2/2)*T$	4.406
Standard deviation of $\ln(S_T) = \sigma*T^{1/2}$	17.68%

99.0% confidence limit on lowest value of	$\ln(S_T)$	3.995
99.0% confidence limit on lowest value of	S_T	54.334

Continuing with the example above: Within a confidence level of 99% - below what value will the asset be in 0.5 years?

With 99.0% confidence asset will be below:	123.674

Continuing with the example above: Within a confidence level of 99% - between which values will the asset be in 0.5 years? [Assume that the 1.0% of values ourside the 99.0% limit are equally distributed above the upper confidence value and below the lower confidence value.]

Between this upper value:	129.250
And this lower value:	51.990

10 Calculating confidence intervals on an asset's future price (alternate formula)

Spot price [S]	[$]	80
Expected return [μ]	[%]	8%
Volatility [σ]	[%]	25%
Future time [T]	[yrs]	0.5
Number sd's	[#]	2.576
Confidence level [p]	[%]	99.0%

Upper limit	129.250
Lower limit	51.990

Click here to see what your answer should look like.

1 Calculating return of a portfolio of assets

A set of assets comprises a portfolio. The returns of each asset together with their portfolio weights are shown below. Find the portfolio return.

Asset	Return	Portfolio weight
# 1	6%	20%
# 2	10%	25%
# 3	11%	55%

Portfolio return	9.75%

2 Calculating volatility of a portfolio of assets

A set of assets comprises a portfolio. The asset covariance matrix is shown below together with each asset's portfolio weights. Find the portfolio volatility.

Asset	Variance / covariance matrix #1	#2	#3	Portfolio weight
# 1	0.010	0.006	0.006	20%
# 2	0.006	0.090	0.024	25%
# 3	0.006	0.024	0.040	55%

Portfolio volatility	16.3%